ERROR ANALYSIS FOR AN IMAGE POINT CORRESPONDENCE ALGORITHM

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ABSTRACT

An Image Point Correspondence (IPC) algorithm enables the determination of 3-D motion parameters of an object from its image sequences. This method is currently being explored for various robotic vision applications, especially those involving motion of video as well as the object under observation. In this paper the sources of error in the motion estimation of objects/scenes are reviewed. The estimate of the error in the determined parameters is developed using a mathematical formulation. Errors in the output are plotted experimentally as a function of the errors in the input.

1 INTRODUCTION

Computation of 3-D motion parameters of a moving object from its image-sequences is an important research area in motion analysis. Motion analysis finds widespread application in robotics and automation, military, space, weather forecasting, medicine, traffic monitoring, segmentation and scene analysis. In the IPC algorithm, 2-D images of a moving object are taken by a camera over an interval of time [1,2,3,4]. The features of the moving object and the correspondence of these features from one image to another are assumed to be known. In addition, the object is assumed to undergo a *rigid-body* motion, which means that the object does not deform over time. The features can be a set of surface points on the object, its edges, vertices, line segments, 2-D moments and moment invariants of its surfaces, or its surface properties like shading characteristics. The 3-D motion parameters to be computed are its attitude, attitude rate, visible surface shape, identification/recognition, and track.

The accuracy of the IPC algorithm depends on the availability of measured parameters. Errors in these input parameters/data result in errors in the estimated motion parameters [5]. Input errors also include the fact that the object has deformed over time. Therefore, the error propagation for various stages of the IPC algorithm will be described in terms of the error bounds.

A background of the IPC algorithm will be given in section 2 . There are various sources of error which are discussed in section 3.1. Definitions of error, relative error and error bounds, along with the error in simultaneous equations will be presented in section 3.2.

2 THE IPC ALGORITHM

In this section, we provide a background of the IPC algorithm as applied to the two-view motion analysis case exclusively. For details the reader is referred to [1]. We consider in detail the equations that track a single point P on a moving object by a moving camera at two different instants of time τ_i and τ_j (where $\tau_j > \tau_i$) respectively. The point P moves from one position P_i to another position P_j due to the rigid-body motion of the object. We assume (R, $\mathbf{T}$) to be the transformation parameters, rotation matrix and translational vector, respectively. Various steps for the formulation of the IPC algorithm are presented in the following discussion:

Step I: The desired relationship between the coordinates of the initial and the final positions of the point P recorded by the camera is given by the following *motion analysis equation*:

$$\mathbf{p}_j = R\ \mathbf{p}_i + \mathbf{T} \tag{1a}$$

such that

$\mathbf{p}_i = (x_i, y_i, z_i)^T$ represents 3-D coordinates of P_i
$\mathbf{p}_j = (x_j, y_j, z_j)^T$ represents 3-D coordinates of P_j

$$R \equiv \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \quad \text{and} \quad \mathbf{T} \equiv \begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix} \tag{1b,c}$$

where $r_{\alpha\beta}$ ($\alpha, \beta = 1,2,3$) are the rotational elements and t_α ($\alpha = 1,2,3$) the translations along x-, y-, and z-axes respectively.

Step II: Define

$$Q \equiv \begin{bmatrix} q_{11} & q_{12} & q_{13} \\ q_{21} & q_{22} & q_{23} \\ q_{31} & q_{32} & q_{33} \end{bmatrix} = \pm \begin{bmatrix} t_3 r_{21} - t_2 r_{31} & t_3 r_{22} - t_2 r_{32} & t_3 r_{23} - t_2 r_{33} \\ t_1 r_{31} - t_3 r_{11} & t_1 r_{32} - t_3 r_{12} & t_1 r_{33} - t_3 r_{13} \\ t_2 r_{11} - t_1 r_{21} & t_2 r_{12} - t_1 r_{22} & t_2 r_{13} - t_1 r_{23} \end{bmatrix} \tag{2a}$$

where Q, the matrix containing *essential elements*, is found from the following homogeneous equation:

$$\mathbf{v_j}^T Q\ \mathbf{v_i} = 0 \tag{2b}$$

where transformations from 3-D object-space coordinates of P_i and P_j to 2-D image coordinates $\mathbf{v_i}$ and $\mathbf{v_j}$ using perspective projections are respectively

$$\mathbf{v_i} = (X_i, Y_i, 1)^T = f\ \mathbf{p_i}/z_i \ ;\mathbf{v_j} = (X_j, Y_j, 1)^T = f\ \mathbf{p_j}/z_j \tag{2c,d}$$

The focal length f of the camera lens is normalized to 1 without any loss of generality. The homogeneous equation in Eq. (2b) is solved for Q from the following set of equations:

$$N\,Q = (-1, -1, \ldots\ldots, -1)^T \tag{2e}$$

where Q has been formed into an 8 x 1 column vector and each element has been divided by q_{33}.
In addition

$$N = (N_1, N_2, N_3, \ldots)^T \tag{2f}$$

$$N_\alpha = (X_\alpha X_\alpha', Y_\alpha X_\alpha', X_\alpha', X_\alpha Y_\alpha', Y_\alpha Y_\alpha', Y_\alpha', X_\alpha, Y_\alpha) \qquad (\alpha = 1, 2, \ldots, n; n \geq 8) \tag{2g}$$

where (X_α, Y_α) are 2-D image coordinates for the α^{th} data point.

Step III: If the singular value decomposition of Q is defined as

$$Q = U \Lambda V^T \tag{3a}$$

where U and V are orthogonal matrices and Λ is a diagonal matrix containing the singular values of Q. There are two solutions for the rotation matrix, R and R', given by

$$R = U\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & \omega \end{bmatrix} V^T \tag{3b}$$

and

$$R' = U\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & \omega \end{bmatrix} V^T \tag{3c}$$

where $\omega = \det(U)/\det(V) = \pm 1$. One of the rotation matrices corresponds to the motion of the object on the same side of the camera (either front or back) and the one to the motion from either front to the back or *vice versa*. We reject the second solution for the rotation matrix corresponding to a situation not encountered in practice.

Also, the translational elements ($\alpha = 1, 2, 3$) up to a scale factor, in terms of essential elements, are given as [2]:

$$t_{\alpha} = -\sqrt{\sum_{\beta=1}^{3}\left(-q_{\alpha\beta}^{2} + q_{\alpha+1,\beta}^{2} + q_{\alpha+2,\beta}^{2}\right)} \tag{3d}$$

where α is cyclic, i.e. $\alpha_4 = \alpha_1$.

3 ERROR ANALYSIS

3.1 SOURCES OF ERROR

Since the IPC algorithm can be implemented using a sequence of object images, any error in the input data and sensor parameters becomes a source of inaccuracy in the output data. The input data is the set of feature coordinates of the object, and the output data are the 3-D motion parameters. The perturbation errors [5] are due to: (i) spurious noise in the sensor and its display, (ii) defocussing, (iii) limited resolution of the camera, (iv) motion blur, (v) thermal instability of light sources, (vi) optical system aberrations, (vii) diffraction effects, (viii) stray light, and (xi) optical properties of the medium.

3.2 DEFINITIONS

The following definition are taken from [6].

3.2.1 ERROR AND RELATIVE ERROR

If N is true value of any parameter and $\hat{N}$ is its approximation, then

$$\text{Error, E} = \text{True Value} - \text{Approximation} = N - \hat{N} \tag{4a}$$

and

$$\text{Relative Error, RE} = \frac{\text{True Value} - \text{Approximation}}{\text{True Value}} = \frac{N - \hat{N}}{N} \tag{4b}$$

3.2.2 ERROR BOUNDS

When any function f(N) with a continuous derivative is evaluated with N replaced by , the relati $\bar{N}$

$$f(N) - f(\bar{N}) = f'(\eta)(N - \bar{N}); \qquad \bar{N} \le \eta \le N \tag{5a}$$

gives us

$$|E(f(N))| \le |f'(\eta)|_{max}\, |E(N)|$$

$$|RE(f(N))| \le \frac{|f'(\eta)|_{max}}{|f(N)|}\, |E(N)|$$

(5b,c)

The maximum error in the product P (where $P = N_1 N_2$) is found to be

$$RE(P) = \frac{N_1 N_2 - \bar{N}_1 \bar{N}_2}{N_1 N_2} = 1 - (1 - RE_1)(1 - RE_2) \tag{5d}$$

i.e.

$$RE(P) = RE_1 + RE_2 - RE_1 RE_2 \tag{5e}$$

where RE(P) refers to P, RE_1 to N_1, and RE_2 to N_2. $|RE(P)|$ is largest when RE_1 and RE_2 are negative. Thus, generally if $P = N_1 N_2 \ldots N_m$,

$$RE(P) = 1 - [(1-RE_1)(1-RE_2)\ldots(1-RE_m)]$$

3.2.3 ERRORS IN SIMULTANEOUS EQUATIONS

In order to investigate errors, we suppose that the set actually solved is

$$\begin{bmatrix} a_{11} & \cdot & \cdot & \cdot & a_{1n} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{n1} & \cdot & \cdot & \cdot & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ \cdot \\ \cdot \\ x_n \end{bmatrix} = \begin{bmatrix} c_1 \\ \cdot \\ \cdot \\ c_n \end{bmatrix} \tag{6a}$$

whereas the *true* values of the coefficients are $a_{\alpha\beta} + \delta a_{\alpha\beta}$, and the *true* values of the right-hand members are $c_\alpha + \delta c_\alpha$. If the true values of the unknowns are denoted by $x_\alpha + \delta x_\alpha$, we see that

$$\begin{bmatrix} a_{11} & \cdot & \cdot & \cdot & a_{1n} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{n1} & \cdot & \cdot & \cdot & a_{nn} \end{bmatrix} \begin{bmatrix} \delta x_1 \\ \cdot \\ \cdot \\ \delta x_n \end{bmatrix} = \begin{bmatrix} \delta c_1 \\ \cdot \\ \cdot \\ \delta c_n \end{bmatrix} - \begin{bmatrix} \delta a_{11} & \cdot & \cdot & \cdot & \delta a_{1n} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \delta a_{n1} & \cdot & \cdot & \cdot & \delta a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ \cdot \\ \cdot \\ x_n \end{bmatrix} \tag{6b}$$

assuming the products of errors, of the form $(\delta a_{\alpha\beta})(\delta x_\beta)$, to be relatively negligible. Thus, if the errors $\delta a_{\alpha\beta}$ and δc_α were known, the solution errors δx_α would be obtained by solving a set of equations which differs from the set actually solved in that the right-hand member c_α is to be replaced by η_α where

$$\eta_\alpha = \delta c_\alpha - (x_1 \delta a_{\alpha 1} + \ldots + x_n \delta a_{\alpha n}) \tag{6c}$$

In practice, the errors $\delta a_{\alpha\beta}$ and δc_{α} do not exceed a certain positive number ε in magnitude, so that . In s $-\varepsilon \le \delta a_{\square\beta}, \delta c_{\square} \le \varepsilon$ rtain only that

$$\left|\eta_{\square}\right| \le E \tag{7a}$$

where

$$E = \left[1 + \left|x_1\right| + \left|x_2\right| + \ldots + \left|x_n\right|\right]\varepsilon \tag{7b}$$

Therefore,

$$\delta x_{\gamma} = \ddot{A}_{\gamma 1}\eta_1 + \ddot{A}_{\gamma 2}\eta_2 + \ldots + \ddot{A}_{\gamma n}\eta_n \qquad (\gamma = 1,2,\ldots,n) \tag{7c}$$

and

$$\left|\delta x_{\gamma}\right| \le \left[\left|\ddot{A}_{\gamma 1}\right| + \left|\ddot{A}_{\gamma 2}\right| + \ldots + \left|\ddot{A}_{\gamma n}\right|\right]E \tag{7d}$$

where $(\alpha = 1,2,\ldots,n)$ are the elements of the γ^{th} row of the inverse of the coefficient matrix. Thus, if the inverse matrix is calculated, approximate upper bounds on the effects of input errors are obtained from the above equation. They are not *strictly* upper bounds. However, they may be accepted as close approximations to true upper bounds.

3.3 ERROR PROPAGATION IN THE IPC ALGORITHM

In this section, we shall analyze the effect of changes in the input parameters on the output parameters for the IPC algorithm. Since various steps are involved in this algorithm, the propagation of the error will be studied and the error bounds found for each stage of the algorithm (Fig. 1). The *two representations* of the rotation matrix will also be considered (see Appendix). First, the reader will be given the background of the IPC algorithm.

3.3.1 ERROR IN ESSENTIAL ELEMENTS

The sensitivity of the essential elements (elements of a matrix Q) to the error in input set of 2-D image coordinates of the features will be studied in this section.

The equation solved is

$$\tilde{N}\tilde{Q} = (-1,-1,\ldots,-1)^T \tag{8a}$$

where $\tilde{N} = N + \delta N$; $\tilde{Q} = Q + \delta Q$ and the true equation to be solved is given by Eq. (2e). Let

$$\delta N_{\square} = (a_{\square 1}, a_{\square 2}, a_{\square 3}, \ldots, a_{\square 8}) \tag{8b}$$

where

$$\delta a_{\square 1} = \delta X_{\square}'X_{\square} + \delta X_{\square}X_{\square}'; \;\; \delta a_{\square 2} = \delta X_{\square}'Y_{\square} + \delta Y_{\square}X_{\square}'; \;\; \delta a_{\square 3} = \delta X_{\square}'; \;\; \delta a_{\square 4} = \delta Y_{\square}'X_{\square} + \delta X_{\square}Y_{\square}';$$

$$\delta a_{\square 5} = \delta Y_{\square}'Y_{\square} + \delta Y_{\square}Y_{\square}'; \;\; \delta a_{\square 6} = \delta Y_{\square}'; \;\; \delta a_{\square 7} = \delta X_{\square} \;\; \text{and} \;\; \delta a_{\square 8} = \delta Y_{\square} \tag{8c}$$

It follows from [6] that

$$N\,\delta Q = -\delta N\,Q = -\begin{bmatrix}\eta_1 \\ \bullet \\ \bullet \\ \eta_n\end{bmatrix} = -\begin{bmatrix}\delta a_{11} & \bullet & \bullet & \delta a_{18} \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \delta a_{n1} & \bullet & \bullet & \delta a_{n8}\end{bmatrix} \tag{8d}$$

where the products of errors are neglected, and (X_α, Y_α) are 2-D image coordinates for the α^{th} data point.

Case I: If N is square (i.e. n = 8) and non-singular,

$$\left|\eta_\alpha\right| \le \varepsilon_a E_a \qquad (\alpha = 1, 2, \ldots, 8) \tag{8e}$$

where

$$E_a = \sum_{\alpha=1}^{3}\sum_{\beta=1}^{3}\left|q_{\alpha\beta}\right| \qquad (\alpha, \beta = 1, 2, 3 \, ; \alpha \text{ and } \beta \ne 3 \Rightarrow q_{33} \text{ not included}) \tag{8f}$$

for $-\varepsilon_a \le \delta a_{\alpha\beta} \le \varepsilon_a \quad (\alpha, \beta = 1, 2, \ldots, 8)$.

Therefore,

$$\left|\delta q_{\alpha\beta}\right| \le E_a \sum_{\gamma=1}^{8}\left|\tilde{A}_{3(\alpha-1)+\beta,\gamma}\right| \qquad (\text{for } \alpha, \beta = 1, 2, 3 \, ; \alpha \text{ and } \beta \ne 3) \tag{8g}$$

where $\tilde{A}_{\alpha\beta}$ ($\alpha, \beta = 1, 2, \ldots, 8$) are the elements of N^{-1}.

Case II: If N is not a square matrix, we define a residual matrix by

$$\delta N^+ = \tilde{N}^+ - N^+ \tag{8h}$$

where N^+ and $\tilde{N}^+$ are the pseudo-inverses of N and $\tilde{N}$ respectively. Then from [7]:

$$\left\|\delta N^+\right\| \le \left\|\delta N_1^+\right\| + \left\|\delta N_2^+\right\| + \left\|\delta N_3^+\right\| \tag{8i}$$

where

$$\left\|\delta N_1^+\right\| \le \left\|\delta N\right\| \cdot \left\|N^+\right\| \cdot \left\|\tilde{N}^+\right\| ; \quad \left\|\delta N_2^+\right\| \le \left\|\delta N\right\| \cdot \left\|\tilde{N}^+\right\|^2 \text{ and } \left\|\delta N_3^+\right\| \le \left\|\delta N\right\| \cdot \left\|N^+\right\|^2 \tag{8j}$$

3.3.2 ERROR IN ROTATIONAL ELEMENTS

The propagation of error in rotational elements due to error in essential elements is investigated in this section.

If the singular value decomposition of $\tilde{Q}$, defined as

$$\tilde{Q} = \tilde{U}\tilde{\Lambda}\tilde{V}^T = (U + \delta U)(\Lambda + \delta\Lambda)(V + \delta V)^T \tag{9a}$$

is an approximation to the true value

$$Q = U \Lambda V^T \tag{9b}$$

where $\delta U, \delta\Lambda, \delta V$ are the error matrices, then the α^{th} singular vector $\tilde{\mathbf{u}}_\alpha$ of the perturbed matrix $\tilde{Q}$, in terms of the α^{th} singular vector $\mathbf{u}_\alpha$ of the matrix Q, can be approximated by the first-order Taylor Series expansion and is given by [8]:

(9c)

Therefore,

$$\tilde{u}_\alpha = u_\alpha + \frac{\partial \tilde{u}_\alpha}{\partial \tilde{q}_{\beta\gamma}}\Big|_{q_\mu = q_\mu} (\tilde{q}_{\beta\gamma} - q_{\beta\gamma}) \qquad (\alpha, \beta, \gamma = 1, 2, 3) \tag{9d}$$

$$\left|\delta u_\alpha\right| \le \left|\frac{\partial \tilde{u}_\alpha}{\partial \tilde{q}_{\beta\gamma}}\Big|_{q_\mu = q_\mu}\right| \left|\delta q_{\beta\gamma}\right|$$

where $q_{\beta\gamma}, \tilde{q}_{\beta\gamma}$ are the entries of the matrices Q and $\tilde{Q}$ respectively. The derivatives of the singular vectors (for κ = 1, 2, ..., 9) are:

$$\frac{\partial \tilde{u}_\alpha}{\partial \tilde{q}_{\beta\gamma}}\Big|_{q_\mu = q_\mu} = \frac{1}{\sigma_{\alpha\alpha}} B_\kappa v_\alpha - \frac{1}{\sigma_{\alpha\alpha}} (u_\alpha^T B_\kappa v_\alpha) u_\alpha + \sum_{\substack{\gamma \ne \alpha \\ \gamma = 1}}^{3} \left[\frac{\sigma_{\gamma\gamma}}{\sigma_{\alpha\alpha}^2 - \sigma_{\gamma\gamma}^2} (u_\alpha^T B_\kappa v_\gamma) + \frac{\sigma_{\gamma\gamma}^2}{\sigma_{\alpha\alpha}\left(\sigma_{\alpha\alpha}^2 - \sigma_{\gamma\gamma}^2\right)} (u_\gamma^T B_\kappa v_\alpha) \right] u_\gamma \tag{9e}$$

$$\left[B_\kappa\right]_{\alpha,\mu} = \begin{cases} 1, & \text{if } \alpha + \mu - 1 = \kappa = 3(\beta - 1) + \gamma \\ 0, & \text{otherwise} \end{cases} \tag{9f}$$

where $\sigma_{\alpha\alpha}$ (α = 1, 2, 3) are the singular values of Q. The error bounds for elements of V have not been computed in this paper.

After finding error bounds for U and V, we are in a position to find the same for the rotational elements, which are given by the following equations:

$$\left|\delta r_{\alpha\beta}\right| \le \varepsilon_{u\sigma v} \sum_{\gamma=1}^{3} \left(\left|u_{\alpha\gamma}\right| + \left|v_{\beta\gamma}\right| \right) \qquad (\alpha, \beta = 1, 2, 3) \tag{9g}$$

where $u_{\alpha\beta}$, $\sigma_{\alpha\beta}$, and $v_{\alpha\beta}$ are the entries of U, Δ , and V respectively;

$\varepsilon_{u\sigma v} = \varepsilon_u = \varepsilon_\sigma = \varepsilon_v$ for $-\varepsilon_u \le \delta u_{\alpha\beta} \le \varepsilon_u$, $-\varepsilon_\sigma \le \delta\sigma_{\alpha\beta} \le \varepsilon_\sigma$, and $-\varepsilon_v \le \delta v_{\alpha\beta} \le \varepsilon_v$

and has been assumed.

3.3.3 ERROR IN TRANSLATIONAL VECTOR

The translational elements (α = 1, 2, 3), in terms of essential elements, are given in Eq. (3d). The error bounds for these elements are given as:

$$\left|\delta t_\alpha\right| \le \varepsilon_q \sum_{\beta=1}^{3} \sum_{\gamma=1}^{3} \left| \frac{q_{\beta\gamma}}{t_\alpha} \right| \tag{10a}$$

for $-\varepsilon_q \le \delta q_{\gamma\kappa} \le \varepsilon_q$ (γ, κ = 1, 2, 3) because

$$\frac{\partial t_\alpha}{\partial q_{\beta\gamma}} = \begin{cases} +\dfrac{q_{\beta\gamma}}{t_\alpha} & \text{for } \beta \neq \alpha \\ -\dfrac{q_{\beta\gamma}}{t_\alpha} & \text{for } \beta = \alpha \end{cases} \tag{10b}$$

3.3.4 ERROR IN 3-D MOTION PARAMETERS

In this section, the errors in the motion parameters, due to errors in rotational elements, are studied separately for *two representations* of rotation matrix R [1,3,4, and Appendix].

3.3.4.1 USING FIRST REPRESENTATION OF ROTATION MATRIX

From the definitions of the directional cosines of an arbitrary axis, and the angle of rotation around this axis in Appendix using the *first representation* of R [1,4], the error bounds are found to be:

$$|\delta v_\alpha| \leq \varepsilon_r \sum_{\substack{\beta,\gamma=1 \\ \beta\neq\gamma}}^{3} \left| \frac{\partial v_\alpha}{\partial r_{\beta\gamma}} \right|$$

and

$$|\delta\theta| \leq \varepsilon_r \sum_{\alpha,\beta=1}^{3} \left| \frac{\partial \theta}{\partial r_{\alpha\beta}} \right| \tag{11a,b}$$

for $-\varepsilon_r \leq \delta r_{\beta\gamma} \leq \varepsilon_r \quad (\alpha,\beta,\gamma = 1,2,3)$. A general expression for the partial derivatives are:

$$\frac{\partial v_\alpha}{\partial r_{\beta\gamma}} = \begin{cases} \pm \dfrac{(r_{\alpha+2,\alpha+1} - r_{\alpha+1,\alpha+2})(r_{\gamma\beta} - r_{\beta\gamma})}{d^3} & \text{for } \beta,\gamma = 1,2,3;\ \beta \neq \gamma \text{ and } \alpha \text{ is cyclic} \\ \pm \dfrac{d^2 - (r_{\gamma\beta} - r_{\beta\gamma})^2}{d^3} & \text{for } \beta,\gamma \neq \alpha;\ \beta,\gamma \text{ in cyclic order (opposite sign for opposite order)} \end{cases} \tag{11c}$$

and

$$\frac{\partial\theta}{\partial r_{\alpha\beta}} = \pm \frac{(r_{\alpha\beta} - r_{\beta\alpha})}{d\sqrt{4 - d^2}} \qquad (\text{for } \alpha = 1,2,3;\ \beta = \alpha + 1;\ \alpha,\beta \text{ are cyclic}) \tag{11d}$$

3.3.4.2 USING SECOND REPRESENTATION OF ROTATION MATRIX

The *second representation* of the rotation matrix R is presented in Appendix [3,4]. The error bounds in this case are:

$$|\delta\theta| \leq \left| \frac{d\theta}{dr_{23}} \right| |\delta r_{23}| \tag{12a}$$

$$|\delta\phi| \le \left|\frac{\partial\phi}{\partial r_{13}}\right| |\delta r_{13}| + \left|\frac{\partial\phi}{\partial r_{33}}\right| |\delta r_{33}| \tag{12b}$$

$$|\delta\psi| \le \left|\frac{\partial\psi}{\partial r_{21}}\right| |\delta r_{21}| + \left|\frac{\partial\psi}{\partial r_{22}}\right| |\delta r_{22}| \tag{12c}$$

where

$$\frac{d\theta}{dr_{23}} = \frac{1}{-\sqrt{1 - r_{23}^{2}}} \tag{12d}$$

$$\frac{\partial\phi}{\partial r_{13}} = -\frac{r_{33}}{1 - r_{23}^{2}}\ ; \qquad \frac{\partial\phi}{\partial r_{33}} = \frac{r_{13}}{1 - r_{23}^{2}} \tag{12e}$$

$$\frac{\partial\psi}{\partial r_{21}} = -\frac{r_{22}}{1 - r_{23}^{2}}\ ; \qquad \frac{\partial\psi}{\partial r_{22}} = \frac{r_{21}}{1 - r_{23}^{2}} \tag{12f}$$

4 EXPERIMENTAL RESULTS

In this section, we present experimental results for the IPC algorithm tested successfully on real data. The various error plots will also be discussed. These plots indicate the relationship between the errors in the input set of data (coordinates of the features) and the errors in the output data (motion parameters).

There are difficulties in determining constants in various inequalities appearing in the error bounds of the preceding section. Therefore, we obtained error estimates experimentally. This was done by perturbing the data and measuring the corresponding errors in the parameters calculated by our algorithms. These errors are sample errors rather than averaged ones.

The error plots for previous experiments are shown in Fig. 2 and Fig. 3. In these plots, the x-axis represents the magnitude of the percent relative errors in the input set of coordinates, and the y-axis indicates the corresponding magnitude of percent relative errors in the motion parameters. A brief discussion of these plots follows:

In the plots of Fig. 2, (*aI*, *bI*, *thI*) represent the error curves for the motion parameters (directional cosines of an arbitrary axis and the angle of rotation around this axis) where first representation of R is used. The reference motion parameters were: $\upsilon_1 = 0.1$, $\upsilon_2 = 0.2$, $\theta = 12^{o}$; and $t_1 = t_2 = t_3 = 1.0$. Similarly, the plots in Fig. 3 (*rI, yI, pI*) represent the error curves for the motion parameters (roll, yaw, and pitch) where second representation of R is used. The reference motion parameters were : Roll = 11^{o}, Yaw = 12^{o}, Pitch = 13^{o}; and $t_1 = 1$, $t_2 = 2$, and $t_3 = 3$.

5 CONCLUSIONS

In this paper, the expressions for error bounds were derived for various stages of the IPC algorithm. The perturbed set of coordinates were experimentally fed as the input to the IPC algorithm, and the errors in the output set of motion parameters were examined in terms of various plots with percentage of modulus of relative errors in input set of coordinates versus percentage of modulus of relative errors in the motion parameters. From these plots, and various others, we have found that a telescopic increase in the output errors is due to the IPC algorithm assuming rigid-body motion of an object. Error in the input set of coordinates results from the fact that the object has deformed over a period of time.

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APPENDIX

THE ROTATION MATRIX

In this appendix, the description and the properties of the rotation matrix R, and the computation of rotation parameters from it has been presented [1,3,4].

Let

$$R \equiv \begin{bmatrix} \mathbf{r}_1 \\ \mathbf{r}_2 \\ \mathbf{r}_3 \end{bmatrix} \equiv \begin{bmatrix} \mathbf{c}_1 & \mathbf{c}_2 & \mathbf{c}_3 \end{bmatrix} \equiv \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \qquad (A.1)$$

where $\mathbf{r}_\alpha$ (α = 1,2,3) is the α^{th} row of R, $\mathbf{c}_\alpha$ (α = 1,2,3) its α^{th} column, and $r_{\alpha\beta}$ (α,β = 1,2,3) are its elements.

A.1 THE PROPERTIES OF ROTATION MATRIX R

The rotation matrix R is a 3x3 orthonormal matrix (i.e., $R^{-1} = R^T$) of the first kind (i.e., det(R) = +1). Its property that the determinant must be unity implies the following constraints:

$$\|\mathbf{r}_1\|^2 = \|\mathbf{r}_2\|^2 = \|\mathbf{r}_3\|^2 = 1 \quad \text{(A.2a)}$$

$$\mathbf{r}_1 = \mathbf{r}_2 \times \mathbf{r}_3 ; \mathbf{r}_2 = \mathbf{r}_3 \times \mathbf{r}_1 ; \mathbf{r}_3 = \mathbf{r}_1 \times \mathbf{r}_2 \quad \text{(A.2b)}$$

$$\mathbf{r}_1 \bullet \mathbf{r}_2 = \mathbf{r}_2 \bullet \mathbf{r}_3 = \mathbf{r}_3 \bullet \mathbf{r}_1 = 0 \quad \text{(A.2c)}$$

Also,

$$\|\mathbf{c}_1\|^2 = \|\mathbf{c}_2\|^2 = \|\mathbf{c}_3\|^2 = 1 \quad \text{(A.2d)}$$

$$\mathbf{c}_1 = \mathbf{c}_2 \times \mathbf{c}_3 ; \mathbf{c}_2 = \mathbf{c}_3 \times \mathbf{c}_1 ; \mathbf{c}_3 = \mathbf{c}_1 \times \mathbf{c}_2 \quad \text{(A.2e)}$$

$$\mathbf{c}_1 \bullet \mathbf{c}_2 = \mathbf{c}_2 \bullet \mathbf{c}_3 = \mathbf{c}_3 \bullet \mathbf{c}_1 = 0 \quad \text{(A.2f)}$$

The equations (A.2a) through (A.2f) can be written as

$$r_{11}^2 + r_{12}^2 + r_{13}^2 = r_{21}^2 + r_{22}^2 + r_{23}^2 = r_{31}^2 + r_{32}^2 + r_{33}^2 = 1$$

$$r_{11}^2 + r_{21}^2 + r_{31}^2 = r_{12}^2 + r_{22}^2 + r_{32}^2 = r_{13}^2 + r_{32}^2 + r_{33}^2 = 1$$

$$r_{11} = r_{22} r_{33} - r_{23} r_{32} ; r_{12} = r_{23} r_{31} - r_{21} r_{33} ; r_{13} = r_{21} r_{32} - r_{22} r_{31} ;$$

$$r_{21} = r_{32} r_{13} - r_{12} r_{33} ; r_{22} = r_{11} r_{33} - r_{13} r_{31} ; r_{23} = r_{12} r_{31} - r_{11} r_{32} ;$$

$$r_{31} = r_{12} r_{23} - r_{13} r_{22} ; r_{32} = r_{13} r_{21} - r_{11} r_{23} ; r_{33} = r_{11} r_{22} - r_{12} r_{21} ;$$

$$r_{11} r_{21} + r_{12} r_{22} + r_{13} r_{23} = r_{21} r_{31} + r_{22} r_{32} + r_{23} r_{33} = r_{31} r_{11} + r_{32} r_{12} + r_{33} r_{13} = 0$$

and

$$r_{11} r_{12} + r_{21} r_{22} + r_{31} r_{32} = r_{12} r_{13} + r_{22} r_{23} + r_{32} r_{33} = r_{13} r_{11} + r_{23} r_{21} + r_{33} r_{31} = 0$$

All of these equations are not independent. Their interdependence can be recognized from the following identity:

$$(r_{11} r_{21} + r_{12} r_{22} + r_{13} r_{23})^2 = (r_{11}^2 + r_{12}^2 + r_{13}^2)(r_{21}^2 + r_{22}^2 + r_{23}^2) - (r_{11} r_{22} - r_{12} r_{21})^2 - (r_{12} r_{23} - r_{13} r_{22})^2 - (r_{13} r_{21} - r_{11} r_{23})^2$$

A.2 THE TWO REPRESENTATIONS OF ROTATION MATRIX R

The rotation matrix R, used extensively in the motion analysis, can be expressed in either of the two representations explained in what follows:

A.2.1 THE FIRST REPRESENTATION

The rotation of an object by an angle θ around an axis, passing through the origin of the frame with which the camera coordinate system aligns, with directional cosines ν_1, ν_2, ν_3, is expressed as a rotation matrix R [1,4], where

$$R = \begin{bmatrix} \nu_1^2 + (1-\nu_1^2)\cos\theta & \nu_1\nu_2(1-\cos\theta) - \nu_3\sin\theta & \nu_1\nu_3(1-\cos\theta) + \nu_2\sin\theta \\ \nu_1\nu_2(1-\cos\theta) + \nu_3\sin\theta & \nu_2^2 + (1-\nu_2^2)\cos\theta & \nu_2\nu_3(1-\cos\theta) - \nu_1\sin\theta \\ \nu_1\nu_3(1-\cos\theta) - \nu_2\sin\theta & \nu_2\nu_3(1-\cos\theta) + \nu_1\sin\theta & \nu_3^2 + (1-\nu_3^2)\cos\theta \end{bmatrix} \tag{A.3}$$

with the additional constraint:

$\nu_1^2 + \nu_2^2 + \nu_3^2 = 1$

A.2.1 THE SECOND REPRESENTATION

The rotation is also expressed in terms of the Euler angles: θ (*roll* or *tilt*), ϕ (*yaw* or *swing*), ψ (*pitch* or *pan*). Here the rotation matrix R is given by [3,4]:

$$R = R_\phi R_\theta R_\psi \tag{A.4a}$$

where R_ψ represents a rotation of angle ψ around the z-axis in the camera coordinate frame, R_θ represents a rotation of angle θ around the new x-axis, and R_ϕ represents the rotation of angle ϕ around the new y-axis. This is one of the several conventions used in this paper - the other conventions being the product of the three matrices in any order. These conventions do not yield the same rotation matrix R, simply because the product of the matrices are non-commutative. The three rotation matrices are:

$$R_\psi = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad ; \quad -\pi \le \psi < +\pi \tag{A.4b}$$

$$R_\theta = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \quad ; \quad -\frac{\pi}{2} \le \theta < +\frac{\pi}{2} \tag{A.4c}$$

and

$$R_\phi = \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix} \quad ; \quad 0 \le \phi < 2\pi \tag{A.4d}$$

The rotational elements in this case are:

$r_{11} = \cos\phi \cos\psi - \sin\psi \sin\phi \sin\theta$

$r_{12} = \cos\phi \sin\psi + \cos\psi \sin\phi \sin\theta$

$r_{13} = -\cos\theta \sin\phi$

$r_{21} = -\sin\psi \cos\theta$

$r_{22} = \cos\psi \cos\theta$

$r_{23} = \sin\theta$

$r_{31} = \cos\psi \sin\phi + \sin\psi \sin\theta \cos\phi$

$r_{32} = \sin\psi \sin\phi - \cos\psi \sin\theta \cos\phi$

$r_{33} = \cos\phi \cos\theta$

A.3 COMPUTATION OF 3-D MOTION PARAMETERS

A.3.1 ATTITUDE

Using first representation of R, we see that

$$\nu_\alpha = \pm \frac{\left(r_{\alpha+2,\alpha+1} - r_{\alpha+1,\alpha+2}\right)}{d} \quad (\alpha = 1,2,3;\ \alpha \text{ is cyclic}) \tag{A.5a}$$

$$\theta = \pm \sin^{-1}\left(\frac{d}{2}\right) \tag{A.5b}$$

where

$$d = \sqrt{\sum_{\substack{\alpha=1 \\ \beta=\alpha+1}}^{3} \left(r_{\beta\alpha} - r_{\alpha\beta}\right)^2} \qquad (\alpha, \beta \text{ are cyclic}) \tag{A.5c}$$

If second representation of R is used, we see that

$$\text{Roll} = \theta = \tan^{-1}\left(\frac{r_{23}}{\sqrt{1 - r_{23}^{2}}}\right) \tag{A.5d}$$

$$\text{Yaw} = \phi = \tan^{-1}\left(\frac{-r_{13}}{r_{33}}\right) \tag{A.5e}$$

and

$$\text{Pitch} = \psi = \tan^{-1}\left(\frac{-r_{21}}{r_{22}}\right) \tag{A.5f}$$

A.3.2 ATTITUDE RATES

The *interframe attitude rates* between the two frames F_i and F_{i+l} at instants τ_i and τ_{i+1} are given as:

angular velocity of rotation, $\omega_\theta = (\theta_{i+1} - \theta_i)/\Delta\tau$ (A.6a)

if the first representation of R is used. If the second representation of R is used, then

roll rate, $\omega_\theta = (\theta_{i+1} - \theta_i)/\Delta\tau$ (A.6b)

yaw rate, $\omega_\phi = (\phi_{i+1} - \phi_i)/\Delta\tau$ (A.6c) *pitch rate*, $\omega_{\psi} = (\psi_{i+1} - \psi_i)/\Delta\tau$ (A.6d)

In Eqs. (A.6a,b,c,d),

$\Delta\tau = \tau_{i+1} - \tau_i$ (A.6e)

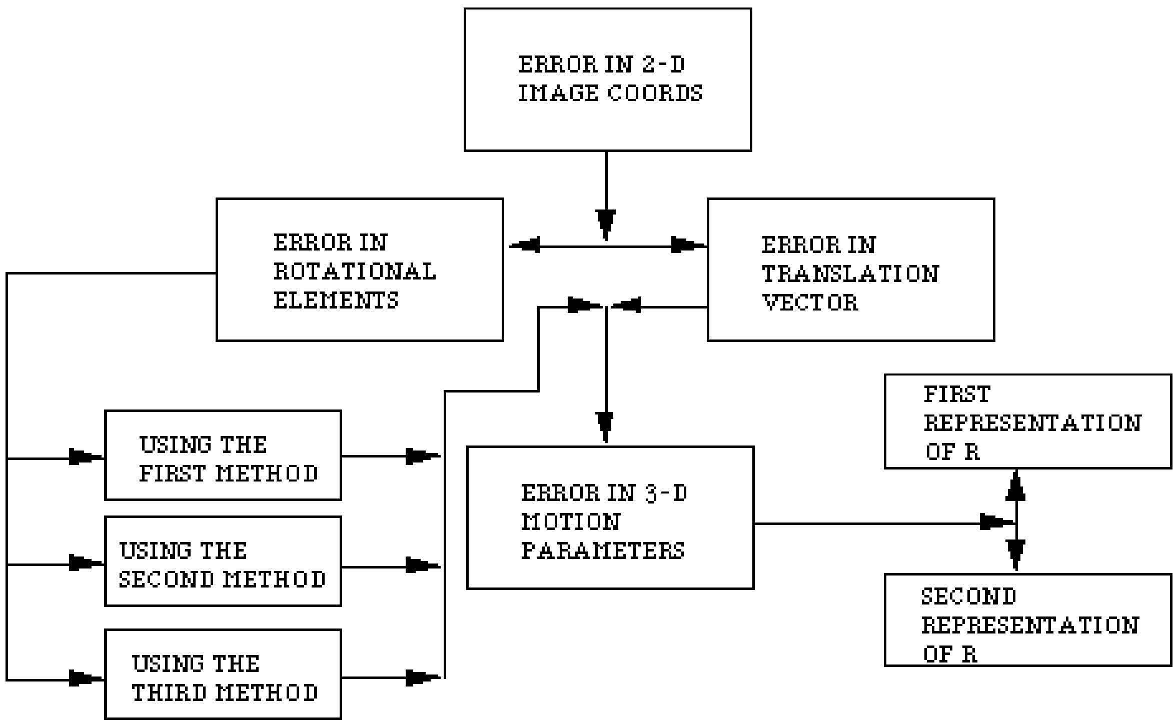

Fig. 1. Error analysis for various stages of the IPC algorithm.

Fig. 2 and Fig. 3 for error plots not available. See Robotics and Expert Systems, Vol. 4, Proceedings of ROBEXS '89, The Fourth Annual Workshop on Robotics and Expert Systems, Hyatt Rickeys Hotel, Palo Alto, California, August 2-4, 1989, pp. 140-141.

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Sunil Fotedar is presently senior associate engineer in the Tacking and Laser Systems Section at Lockheed Engineering and Sciences Company (LESC), Houston, Texas. He received B.E. degree in Electrical Engineering from Regional Engineering College, Srinagar, India; and M.S. degree in Electrical and Computer Engineering from Rice University, Houston, Texas.

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